Gridless Beats

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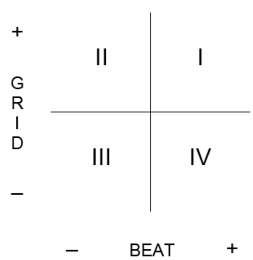
Jazz music conflates two seeming opposites: the fixed foundation of a regular beat and the fluidity of microrhythmic variation. By fuzzying subdivision values and slicing beats asymmetrically, jazz musicians enrich the monotony of steady beat patterns with an elastic treatment of time. Because such rhythmic sophistication derives from spontaneous invention and not from the directives of music notation, a composer wishing to emulate jazz rubatos faces an interesting technical challenge. In this article I look at how certain microrhythmic properties of jazz can be simulated using standard music notation. My goal is to bridge aspects of jazz improvisation and contemporary composition by providing insights into the former and toolsets for the latter.

Grids and Beats

The beat (or tactus) is often theorized as one of several isochronous strands within a temporal hierarchy (Lerdahl and Jackendoff 1983, Yeston 1976). The vertical julienning of horizontal time calls to mind a grid in which the smallest unit assembles rhythm and meter from the bottom up. One could think of both the grid and the beat as non-sounding structural concepts, as in music where these are imagined by the performer and/or notated by the composer but not conveyed aurally to the listener.[1] In this discussion, grids and beats are understood as perceptually salient and mutually interacting.

The matrix in Example 1 shows four different types of grid/beat interaction. It is almost always the case that whenever a beat is present, so is its grid. "If beat, then grid" is the premise of quadrant I interactions. "Stayin' Alive" falls in this quadrant, as does Schoenberg's *Klavierstücke* op. 33. Or, the grid can function as an audible frame onto which attack points are pinned without engendering a convincing sense of tactus, as in quadrant II's beatless grid. Ligeti's "Désordre" and "L'escalier du diable" are good examples. Or, one could do away with the grid altogether, an action that dissolves the beat as well. Quadrant III's "no grid, no beat" is a frequent result of graphic scores, pointillism, chance, extreme slowness, Impressionistic timelessness, and rhythmic saturation.

Gridless beats fall into quadrant IV. A gridless beat is a beat containing onsets that are not aligned with its isochronous subdivisions. Hence gridless beats are in violation of Lerdahl and Jackendoff's first Metrical Well-Formedness Rule, which states that onsets "must be associated with a beat [i.e., subdivision pulse] at the smallest metrical level present at that point in the piece" (1983, 72). To this we add the perceptual dimension contained in London's first metric Well-Formedness Constraint, which states that subdivision interonset intervals "must be at least ≈ 100 ms" (London 2004, 72).[2]



EXAMPLE 1: FOUR TYPES OF GRID/BEAT INTERACTION

Gridlessness resembles microtonality's use of pitches not on the equal-tempered lattice. Although it is not my wish to embrace pitch/rhythm isomorphisms, the idea that rhythmic dissonance can ensue given non-categorizable temporal ratios parallels that of harmonic dissonance given non-equal-tempered harmonic ratios. John Adams touches on this analogy when discussing the role of the grid in his compositions (Adams et al. 1996, 96):

> In the creation of temporal relationships there seems to be a point beyond which musical time itself begins to sound dissonant. This is a funny way of describing things, but I feel that there's a natural way of dividing time and then there's a dissonant way of dividing time. I don't know where the dissonance begins. Two against three obviously does not feel dissonant to me. . . . Three against five begins to push the envelope.

Paul Berliner sums up this article's main gist by noting that "efforts to capture the complexity of jazz despite the limitations of notation often result in dense representations . . . that are difficult for performers to interpret" (1994, 158). The following pages illustrate how jazz musicians bend the grid's vertical and horizontal coordinates to dissociate the beat-level from the other layers in the rhythmic hierarchy. I discuss five different forms of gridlessness: displacement, swing, acceleration, polyrhythm, and wavering. Even thought each of these categories also exists in gridded form, our interest will lie in whether—and if so, how—their gridless form can be simulated using standard music notation.

Displacement

Also known as phase shifting, displacement moves the grid of a rhythmic figure with respect to the grid of the accompaniment. Example 2 shows gridded and gridless displacements. Only in the former type are the melodic grid's subdivisions aligned over the accompaniment's. Gridded jazz displacements have been discussed elsewhere (e.g., Waters 1996); they are generally easy to notate and require no additional words here. Notating gridless displacements is another matter.

The graph in Example 3 gives timing information for a ballad phrase by Chet Baker.[3] Each point represents a note onset; *x* and *y* coordinates plot time of onset and interonset interval (IOI), respectively.[4] Horizontal coordinates denote metronomic subdivision values for the triplet, sixteenth note, and quintuplet. Time is demarcated by vertical coordinates along the *x*-axis: tall coordinates correspond to beat onsets in the

EXAMPLE 2: A GRIDDED AND A GRIDLESS DISPLACEMENT

accompaniment and short coordinates correspond to subdivisions (sixteenths in this example).[5] Each onset includes an error tail displaying the magnitude of displacement from the subdivision. In this example, displacements are small in the beginning and quickly grow to about the size of a sixteenth note.

A gridless beat contains points between adjacent vertical coordinates. This example abounds with them.[6] For an onset to land between vertical coordinates, the preceding onset must have lay between adjacent horizontal coordinates. In Baker's case, the IOIs in the second and third beats are slower (higher) than the sixteenth note and faster (lower) than the triplet, which nudges the onset times towards the middle of the vertical subdivision lines. Therefore gridlessness is a product of note duration and note placement. Pragmatically, this means that we can ballpark gridlessness by looking at the graph's *y*-axis fluctuation as well as its *x*-axis intra-column density.

EXAMPLE 3: CHET BAKER, TRUMPET: "AUTUMN IN NEW YORK" (56 BPM, 3:33)

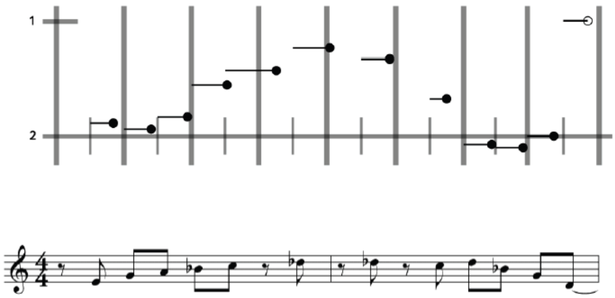
There is no easy way to simulate the temporal richness of Baker's passage with standard music notation. Two approximations appear in Example 4. In system (a), we make an adjustment on beat two, where the delays emerge in earnest, and from that point on make all notes late by one sixteenth. But now we have a gridded displacement; the delays in the original phrase are almost a sixteenth wide, not a whole sixteenth. In system (b), we displace onsets by a smaller margin such as a thirty-second note. There are three strikes against this alternative. First, this version is also gridded because the tempo is slow enough to permit thirty-second note subdivisions (roughly 130 ms). Second, the notation looks strained and would likely result in a rigid performance. It would be easier—albeit indifferent to our present goals—to notate the phrase normally and include a "laid back" directive. Third, the *y*-axis fluctuation we saw in the timing graph is completely absent from this isochronous version.

A faster phrase by another trumpeter appears in Example 5.[7] Even more than in the previous example, Art Farmer's attacks are so far displaced as to appear gridded onto the subsequent subdivision coordinate. The alternate transcription in Example 6 takes this observation into account. The new graph retains the original onset times (as performed) while redrawing the subdivision coordinates to match those of the new transcription. This interpretation contains fewer and smaller displacement tails—a victory for the *x*-axis. But the smooth bell-curve charted by the gridless *y*-axis still eludes us.

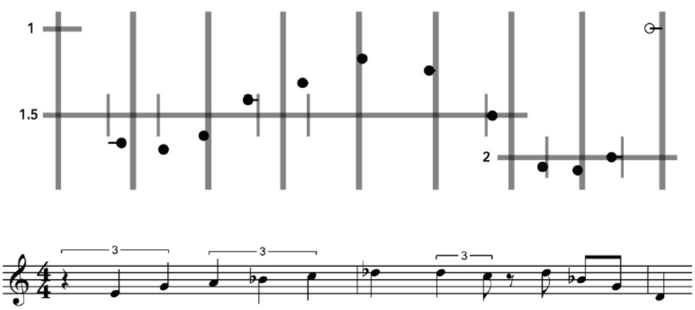
(a)

(b)

EXAMPLE 4: NEITHER NOTATION DOES THE TRICK



EXAMPLE 5: ART FARMER, TRUMPET "BLUES FOR PABLO" (140 BPM, 2:21)



EXAMPLE 6: QUANTIZED VERSION OF EXAMPLE 5

The upshot of the two excerpts just introduced is that notated simulations of gridless displacements tend to fall short. The main challenge seems to lie in replicating *x*- and *y*-coordinate gridlessness when these are presented consecutively and en masse. We will see later how these two features can be simulated in other scenarios.

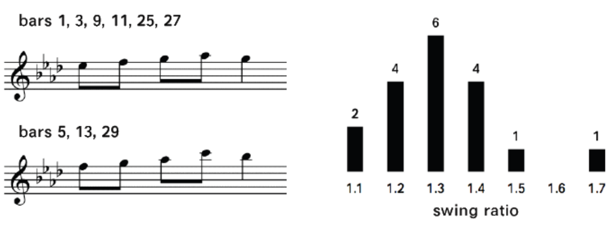
SWING

Jazz eighth-notes usually follow a long–short pattern. The exact proportion between beat-eighth and upbeat-eighth varies considerably depending on context. The tripleted long–short ratio of 2–to–1 is gridded at most tempos but rare in the melodic line (Benadon 2006, Friberg and Sundström 2002). The more frequent ratios are ungridded. A "swing ratio" of the form *x:y* requires *x+y* subdivisions of the beat. This sum, call it *s*, quickly puts us in ungridded territory: if *b* equals beat size, then b/s < 100 ms at most tempos.

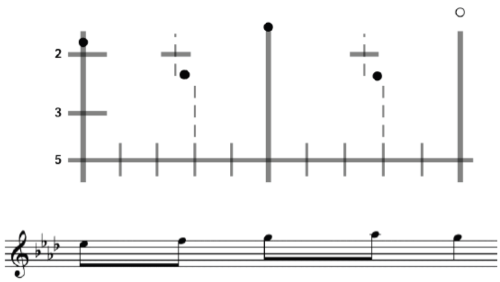
At $s = 5$ we have 3:2 (or 1.5), a standard swing ratio. But it is gridless above 120 bpm (b = 500 ms), which is a fairly slow tempo for jazz. At $s = 7$ we have 4:3 (1.33), also a standard ratio; it is gridless above 86 bpm. Continuing is fruitless: grid-friendly tempos continue to drop as the value of *s* increases.[8]

Consider Thelonious Monk's rendition of "I'm Confessin' (That I Love You)."[9] The melody contains strings of eighth-notes in the first, third, and fifth measures of every A section of the 32-bar AABA. The histogram in Example 7 shows that Monk plays with a consistent inflection of roughly 1.3. The two representative beats in Example 8 contain ratios of 1.2 and 1.3, respectively. At this tempo, the best we can do is provide quintuplet subdivisions for a 3:2 split of the beat. This 1.5 ratio approximates Monk's preferred values well enough, as the long dashed line shows, but its feel is appreciably different from Monk's.

A practical, or rather impractical ramification of the two above paragraphs is that most swing ratios cannot be simulated with music notation.



EXAMPLE 7: THELONIOUS MONK, PIANO: "I'M CONFESSIN' (THAT I LOVE YOU)" (103 BPM)



EXAMPLE 8: MONK, PIANO: "I'M CONFESSIN' (THAT I LOVE YOU)" (0:56)

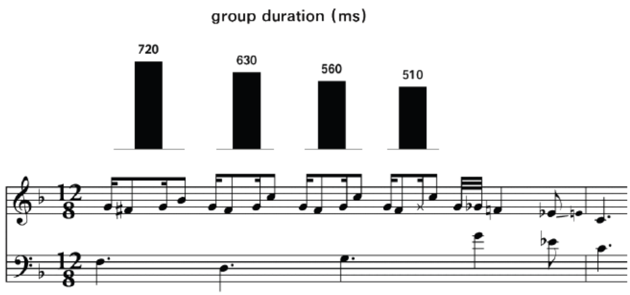
Acceleration

Unlike the standard accelerando which voraciously consumes all voices, the jazz accelerando leaves out the accompaniment.[10] In Example 9, the tempo remains steady while Kenny Drew's melodic line speeds up in gridless fashion.[11] Standard notation can imitate this effect by increasing the melodic tuplet count from beat to beat, recruiting finer and finer subdivision gradations. But such a gridded approach can sound bumpy, for it risks exposing the sudden transitions between subdivision categories. We would not be far off the mark if we notated Drew's accelerando with a beat of triplets followed by a beat of quintuplets and a beat of septuplets (or sextuplets—at this rate their difference is marginal). We would, however, lose the smooth descent, which outlines a fairly continuous trajectory from slow to fast. We would also obscure the phrasing of the four-note group, which Drew cycled with increasing speed. The same could be said of Johnny Hodges's looped accelerating pattern in Example 10.[12]

We can simulate gridless acceleration with a little sleight of notation. Example 11 contains an accelerating rhythm in the melody. The use of spatial notation provides a good guide as to when the onsets should occur, but this approach has limitations. John MacIvor Perkins has noted that it can lead to "gross inaccuracies in performance" (1965, 53), especially if we wish to orchestrate the phrase and potentially place concerted accents over certain notes.

The same music is re-notated in Example 12. This passage accelerates from 60 to 150 bpm, a more than two-fold increase of 2–to–5. The

EXAMPLE 9: KENNY DREW, PIANO; PAUL CHAMBERS, BASS: “I’M OLD FASHIONED” (80 BPM, 4:24)



EXAMPLE 10: JOHNNY HODGES, ALTO SAXOPHONE; RAY BROWN, BASS: “FUNKY BLUES” (70 BPM, 1:41)

EXAMPLE 11: ACCELERATING WITH SPATIAL NOTATION

melodic line is affected by the sizeable tempo change as much as it was with the spatial configuration. We perceive the accompaniment as stable (as it was in the original version) because its augmenting note values compensate for the accelerando on the fly. We accomplish this by having the tactus gradually morph its written value from a quarter-note up to a half-note tied to an eighth-note—that is, until it stands in a 5:2 ratio to its original quarter-note value. In this way, the acceleration is perceivable only in the melodic line, which the listener interprets as a dynamic transformation over a steady, global tempo. Different configurations can be devised in addition to the one presented here, provided that the heard speed of the accompaniment is the same before and after the accelerando. For instance, if the tempo doubles, quarter-notes become half-notes via step-by-step augmentation. Also, the overall span of the acceleration can be made shorter or longer than the one in this example, which lasts four (perceived) beats.

EXAMPLE 12: ACCELERATING WITH GRID ADJUSTMENT

Polyrhythm

Superimposing two non-integer related grids yields a polyrhythm. In Example 13, drummer Tony Williams re-groups the triplets of his ride cymbal into fours (the hi-hat pedal is notated with an x). This results in a 3–to–4 overlay that seems to slow the tempo from 128 to 96 bpm. The effect is striking because of Williams's rich phrasing and the ensemble's immediate fine-tuning to his metric game. Expressive complexity aside, the two streams share the same speed and their composite result is therefore gridded.

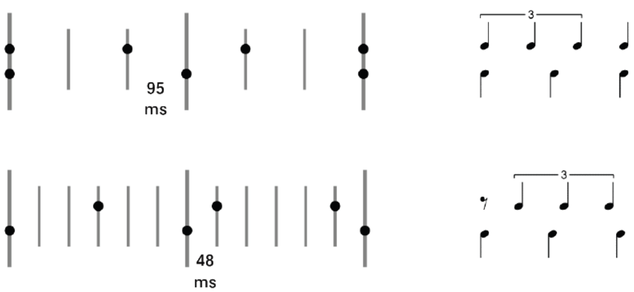
EXAMPLE 13: TONY WILLIAMS, DRUMS: "PEE WEE" (128 BPM, 2:30)

In order for a grid to participate in the formation of a well-formed polyrhythm, its subdivisions must be no faster than the metrically admissible value of 100 ms. A gridless polyrhythm results when the fusion of its constituent grids produces subdivisions smaller than this value—that is, if it contains two grids with a subdivision speed relation of x/y and $b/(x{*}y) < 100$, where b is the ms duration span between alignment points in the polyrhythm. Put yet another way: gridlessness occurs when the distance between one grid's subdivision coordinate and its closest subdivision coordinate in the other grid is smaller than 100 ms and greater than zero.[13]

The passage in Example 14 contains two polyrhythms.[14] The second polyrhythm stretches eight eighth notes into the place of nine, allowing Booker Little to resynchronize with the bass on the last beat of the last measure. This polyrhythm is unquestionably gridless. For it not to be, the tempo would have to be a glacial 38 bpm. The first polyrhythm, which repeats three times, is also gridless but less blatantly so.[15] Had the triplet group begun on the beat, it would have been just barely gridded. But the triplet group begins on the upbeat, and that changes everything. The composite grid now requires twice as many subdivisions, as Example 15 illustrates.

These observations suggest that gridless polyrhythms need not be notationally complex. One could of course devise a multifarious tuplet to demonstrate the notational intricacy of a certain gridless polyrhythm. My goal is to show how gridlessness can be attained with relatively simple notation, rather than to show this for every conceivably gridless rhythm. In any case, readers thirsty for more grid superposition need simply read on.

EXAMPLE 14: BOOKER LITTLE, TRUMPET; RON CARTER, BASS: "FAR CRY"
(214 BPM, 1:03)



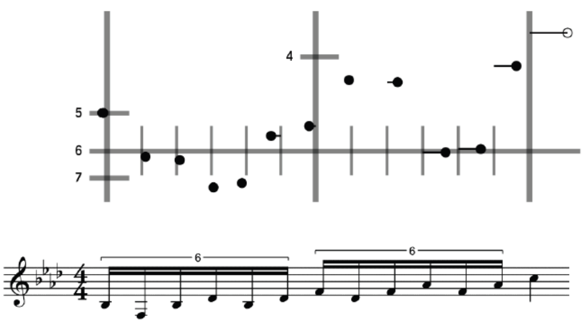
EXAMPLE 15: ON-THE-BEAT (GRIDDED) AND OFF-THE-BEAT (GRIDLESS)
TRIPLETS AT 210 BPM

Wavering

We saw earlier how nominally equal subdivision values can waver in actual duration. The excerpts in Examples 16 and 17 show vacillating sixteenth notes and sextuplets, respectively.[16] Louis Armstrong's celebrated phrase consists of a descending pentatonic tetrachord that produces, in the words of Gunther Schuller, an "almost stammering repetitive phrase that seems to float, completely unencumbered rhythmically, above the accompaniment" (1968, 119). John Coltrane's wavering sextuplets create delays leading up to and including the third beat of the measure. In both excerpts, few dots fall on a horizontal coordinate. These rhythms give the impression of a tempo-knob being turned suddenly to the left and right in order to dispel any sense of periodicity.[17]

Suppose that we wish to notate the Armstrong passage with the intention of hearing it re-created by a performer or group. We might

EXAMPLE 16: LOUIS ARMSTRONG, TRUMPET: "WEST END BLUES" (83 BPM, 2:45)



EXAMPLE 17: JOHN COLTRANE, TENOR SAXOPHONE: "LIKE SOMEONE IN LOVE" (58 BPM, 2:42)

opt for the sixteenth note but think of it as a quantized version of the real rhythm by indicating above the score that the phrase be played "unpredictably" (or "*à la* Satchmo"). Such an approach will ensure temporal deviations of one type or another, but maintaining a sense of a steady beat while spontaneously varying the note durations is a skill that may be expected of certain performers only, especially those well steeped in improvisation. Moreover, this approach lacks the kind of precision required for the phrase to be performed simultaneously by more than one player. In light of these drawbacks, we could try notating the durations as accurately as possible. However, notating complex rhythms with maximum exactitude constitutes a slippery slope. Our goal is to attain sufficient rhythmic complexity while retaining a considerable degree of notational simplicity.[18]

We can use tempo substitutions to loosen gridded notated rhythms with a dose of artificial expressive timing. Example 18 shows a simple melody that conforms mostly to a quadruple grid. An alternate representation appears in Example 19, where the new governing tempo of 84 bpm stands in a 7:8 ratio to the original one of 96 bpm. To create the illusion of 96 bpm, the accompaniment has been re-notated using the note value of a quarter note tied to a dotted eighth note, for a total of seven sixteenth notes rather than the eight contained by a half-note in the 96-bpm version. This means that each half note, as perceived, is configured to accommodate a septuplet. A glance at Example 20 provides one reason for avoiding septuplet notation in the new version. As stated earlier, the simulation is designed to maximize rhythmic flexibility while reducing notational complexity. The motivation is purely practical and carries no aesthetic bias.

The hidden tempo technique notates the music in a different tempo than the one that is actually perceived. Since the heard beats follow a different timeline from that of the melody, this dichotomy brings about gridless wavering.[19] Example 21 gives timing data for the new 84-bpm

EXAMPLE 18: A GRIDDED MELODY

EXAMPLE 19: GRIDLESSNESS BY HIDDEN TEMPO

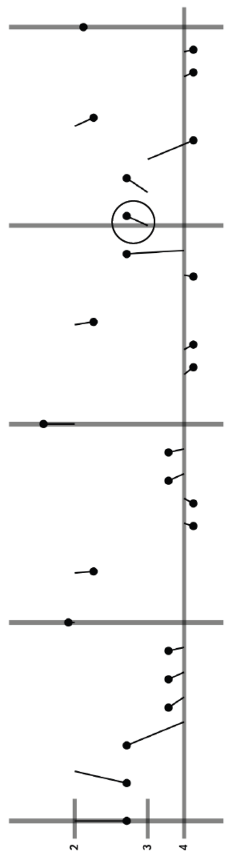
EXAMPLE 20: YOU WANT A PIECE OF ME?

melody over the original 96-bpm grid. Error tails point to the location of the original gridded onsets. Vertical wavering pays horizontal dividends. For instance, note the placement of the high D, which is circled in the graph. This attack comes 60 ms after the beat, a perceivable margin that would be difficult to express using the original tempo's notation (close to a decuplet). Our earlier inability to simulate displacements is modestly mitigated here.

More on Grid Superposition

Grid superposition has taken many forms in the literature: as metric dissonance (Cohn 1992, Krebs 1999, Yeston 1976), as tempo substitutions (Benadon 2009, Huang & Huang 1994), as oscillators phase-locked to external events (Large and Palmer 2002), and as polyrhythm in jazz (Folio 1995), metal (Pieslak 2007), and West African drumming (Locke 1982).

The idea of concurrent speeds can also shape compositional design (Mead 2007), most famously in the music of Elliott Carter and Conlon Nancarrow. Without delving into the specifics of their individual



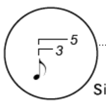
EXAMPLE 21: ONSETS OF EXAMPLE 19 OVER GRID OF EXAMPLE 18

techniques,[20] I pause briefly to point out two main differences between their approach and the examples that appear below. First, rarely does either Carter or Nancarrow establish a solid sense of steady beat. Instead, there is an overall democratization in the perceptual salience of pulse streams, which are themselves often in a state of flux. One classic exception is the first movement of Carter's "Cello Sonata," where the piano maintains a steady quarter-note pulse. Another exception is Nancarrow's "Study no. 27," dubbed the "Ontological Clock" study. Kyle Gann describes how the unchanging ostinato "provides a perceptual yardstick against which all the accelerating and decelerating voices are heard" (1995, 160). A second difference is that in Carter's music (and to a lesser extent in Nancarrow's, to the extent that it employs standard music notation), different speeds are established by particular note values (or summations thereof) rather than by grids containing hierarchized collections of note values, as is the case here.

The reason why hidden tempos lead to gridlessness is that the melodic grid projects new subdivisions onto the grid supplying the perceived beat. The table in Example 22 compares durational values of beat subdivisions belonging to two tempos that are related by a 5:4 ratio.[21] This layout can help answer questions such as: How does one notate laid back sixteenth notes at 80 bpm? As an alternative to the eighth note triplet nested in an eighth note quintuplet, one could try an eighth note triplet at 100 bpm. Rather than comparing absolute durations, we might think in terms of relative ratio relationships. Given any tempo, we can measure the speed of a subdivision value in terms of its ratio relation to the beat (let us assume the quarter note). For instance, the ratio of the eighth note to the quarter note is 2:1. The impression of gridlessness involves tapping into the kind of complex ratios not offered by simple note values. If two tempos are related by a ratio $R = a/b$ ($a<b$), the relationship r between a subdivision s of tempo a and the beat rate of tempo b is given by $r(b) = s(a)*R$.

Let us illustrate this formula by revisiting the identity mentioned above, where one tempo's eighth note triplet was deemed comparable to a slow sixteenth note at another tempo. In that case, $R = 4/5$, $r = 3$, and $s = 15/4$, a fraction that can be notated in the slower tempo, but not without orthographic laboriousness. Example 23 contains similar comparisons. To constrain the number of possible outcomes in this illustration, I include only three values of s (eighth note triplet, sixteenth note, and sixteenth note quintuplet) and consequently only three values of R.

A fragment from my composition "Song 72" for alto saxophone and piano appears in Example 24.[22] The ratio between the two instruments'

80 bpm	ms	ms	100 bpm
Quarter-note	750		
		600	Quarter-note
Dotted eighth-note	563		
Quarter-note triplet	500		
		450	Dotted eighth-note
		400	Quarter-note triplet
Eighth-note	375		
Eighth-note quintuplet	300	300	Eighth-note
Eighth-note triplet	250		
		240	Eighth-note quintuplet
	(200)	200	Eighth-note triplet
Sixteenth-note	188		
Sixteenth-note quintuplet	150	150	Sixteenth-note
		120	Sixteenth-note quintuplet

EXAMPLE 22: NOTE-VALUE COMPARISON

tempos is 2/3—72 bpm in the sax accompaniment and 108 bpm in the piano's melodic line. The melody is perceived as gridless because its eighth-note triplets sound like slow sixteenths, a subdivision rate of 9/2.

As these examples demonstrate, superimposing an alternate grid over the prevailing one leads to a blurry collection of beat subdivision values. While this provides gridless variety to the passage, the scope of our new subdivision palette is nonetheless equally limited—we simply replaced one set of exclusively gridded attacks by another equally sized set containing some gridless options. It is possible to brush off this predicament by sequencing tempo modulations that regularly refresh the ratio relationships. During this process, the melodic grid is modulated while the accompaniment remains unchanged.

R	s	r	sounds like . . .
3 : 4	3	9 : 4	a fast eighth
	4	4 : 1	(just a sixteenth)
	5	15 : 4	a slow sixteenth
3 : 5	3	9 : 5	a slow eighth
	4	12 : 5	a slow eighth-note quintuplet (between eighth and triplet)
	5	3 : 1	(just a triplet)
4 : 5	3	12 : 5	a slow eighth-note quintuplet (between eighth and triplet)
	4	16 : 5	a fast eighth-note triplet
	5	4 : 1	(just a sixteenth)

EXAMPLE 23: TEMPO MODULATIONS MAKE GRIDLESS BEATS

EXAMPLE 24: FERNANDO BENADON: "SONG 72"

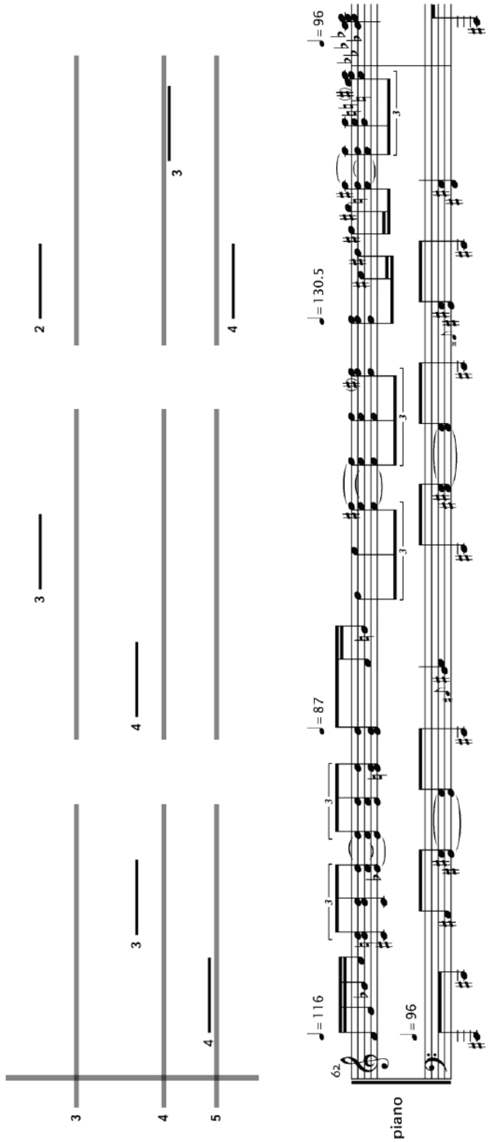
In Example 25, the right hand passes through three new tempos before returning to the original tempo, which remained unperturbed in the left hand. The two parts' beats go out of phase for the duration of the modulations, although only the left hand's beat is felt as a frame of reference. The graph shows how the changing tempos supply new gridless subdivisions to the underlying tempo. This technique need not be restricted to solo piano; I have used it successfully in ensemble works. When the work is conducted, the accompaniment plays unconducted from the point of divergence to the point of convergence while the modulating instruments follow the conductor's shifting tempos. Care must be taken, though: if there are too many modulations in the sequence, and/or if these are too lengthy, the potential for misalignment at the point of desired convergence will be high because of performance error. One way to minimize re-synchronization error due to drift is by choosing simple-enough modulatory pairs at each seam, such as "triplet becomes sixteenth."

Computationally, the challenge lies in having both parts re-synchronize at the right moment. The modulations should be calculated in such a way that—when performed accurately—they amount to an equal temporal span as the non-modulating accompaniment. For a passage with *n* modulation segments, we can determine which modulation combinations yield the correct temporal span using the formula

$$| (B/T) - \Sigma_n(b/t)_i | \leq \epsilon$$

where *B* is the total number of beats in the accompaniment during modulations, *T* is the accompaniment tempo in beats per minute, and *b* is the number of beats in the melody within each modulation segment *i* with tempo *t*. The variable ϵ stands for the acceptable divergence error threshold in seconds—that is, the margin by which the chain of modulations will anticipate or overshoot the desired point of convergence if perfectly executed. It is up to the composer to determine what kind of error threshold is acceptable. Discrepancies in the range of about one tenth of a second are probably harmless; larger ones are likely to be noticeable and they might disrupt re-synchronization.

Simply because a modulation equivalence is mathematically feasible does not mean that it is performatively feasible, especially when the relational disconnect between adjacent tempos renders the modulation impractical.[23] The table in Example 26 provides a list of possible modulation trajectories given a limited set of modulation ratios. These chains contain two modulations; multiple-step chains can be designed with the help of the above formula. This table tolerates convergence discrepancies of up to 100 ms given an accompaniment tempo of 84 bpm. Since



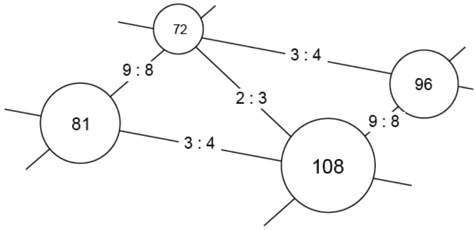
EXAMPLE 25: FERNANDO BENADON: “BÚGI WÚGI”

Beat Span	Tempo Ratio (Accompaniment : Melody)								ε
	2:3	3:4	4:5	3:5	5:3	5:4	4:3	3:2	
	2		2						*− 48*
3		2	2						*+ 71*
	2			3					*+ 95*
	2						2		*0*
		2				2			*0*
			2	4					*0*
	4			2					*− 95*
4				2			2		*− 95*
		2	3						*− 71*
		2		4					*− 71*
		3		3					*+ 36*
			2			2			*+ 71*
	4		3						*+ 48*
	4			4					*+ 48*
			3				2		*+ 48*
				4			2		*+ 48*
	2	5							*+ 60*
	2					3			*+ 60*
		2		6					*+ 71*
	5			3					*+ 95*
				3	2				*+ 95*
	3	4							*0*
5	3			5					*0*
	3							2	*0*
			4	3					*0*
		5		2					*− 36*
				2		3			*− 36*
	5		2						*− 48*
	2			6					*− 48*
			2		2				*− 48*
	4	3							*− 60*
		3					2		*− 60*
			3			2			*− 71*
				4		2			*− 71*
		6	2						*+ 71*
			2					3	*+ 71*
				6		2			*+ 71*
	2		6						*+ 95*
	3		5						*0*
	4				2				*0*
	5						2		*0*
	3						3		*0*
6		4		5					*0*
		3				3			*0*
		4						2	*0*
		2						3	*0*
			6	2					*0*
			3	6					*0*
				5				2	*0*
					2		2		*0*
	4		4						*− 95*
			4				2		*− 95*

EXAMPLE 26: TWO-TEMPO MODULATION CHAINS WITH ϵ VALUES (MS) FOR 84 BPM

this error varies in inverse proportion to tempo, the figures in the last column will be smaller given a tempo that is faster than 84 bpm.

Returning to "West End Blues" (cf. Example 16), we find that the passage can be modeled as a chain of tempo modulations. Assigning a tempo value to the total duration of each four-note group gives us roughly 82, 77, 94, and 83 bpm (730, 780, 640, and 720 ms from B♭ to B♭). The network in Example 27 gives an idea of how these tempos may be approximated proportionally. I do not claim that Armstrong's rhythmic repertory contains tempo modulations, *pace* Schuller's (1968, 117) hearing of a 3:2 "metric modulation" in this same solo's opening cadenza. Rather, my intention is to underscore the feasibility of using tempo modulations to model gridless beats.



EXAMPLE 27: TEMPO NETWORK IN "WEST END BLUES"

Afterword

The beat is the temporal epicenter of countless Western musical traditions. One explanation for the beat's primacy is often framed from a biological standpoint. The beat constitutes the most basic form of entrainment, the process by which neural networks in the brain synchronize their firing pattern with the timing pattern of a regularly recurring external stimulus. Via entrainment, the beat may serve an evolutionary purpose by helping to fine-tune and optimize the brain's temporal structures. Neurons fire in synchrony when one performs a learned task, whereas neural activation during unfamiliar tasks is not well synchronized. And gridded rhythms with clear beats elicit higher activity in the brain's motor areas than rhythms with less clear periodicities (Grahn and Brett 2007). The nexus between the motor system and entrainment mechanisms suggests that the beat's role as a catalyst for conscious physical motion (such as foot tapping) is complemented by an equally active subconscious function. This beat/motion coupling may inform our negotiation of space and is a key component of movement disorder rehabilitation (Thaut 2005, 39–59).

The grid piggybacks on the beat in the form of a hierarchy that is most often metrically parsed. The reason for this, most likely, has to do with subjective rhtyhmization (also known as subjective accentuation), the predisposition to mentally organize isochronous sequences of equal tones into groups of twos or fours (and sometimes threes). This process is observable with neurophysiological methods. Brochard et al. (2003) measured listeners' brain responses to isochronous sequences and found that a slight decrease in amplitude elicited a stronger response when it occurred on a tone whose position was subjectively strong (i.e., position 9 instead of 10 within a binary meter).[24] Once formed, hierarchical structures enhance such interdependent cognitive processes as prediction, attention, clock induction, coding, categorization, grouping, recall, production, and syntax.[25] Tellingly, knowledge of hierarchies is both learned and innate. Experimental evidence shows that sensitivity to temporal hierarchies is as present in adults as it is in children and infants (Bergeson and Trehub 2006). It is no wonder, then, that the grid plays a strong unifying role in such a wide range of musics, including rhythmically complex ones that employ polyrhythmic or "odd-metered" rhythmic textures. This apparent ubiquity may seem curious, for in practice it is not difficult to create gridless beats. Anyone can make such music with only two parts: let one part have a basic pulse, let a second part generate a random series of durations within a prescribed range, and *voilà*. Yet, musical examples that exhibit this kind of temporal dichotomy in beat-based environments are far outnumbered by those of the gridded kind, in which the grid ensconces the beat.

A specific advantage of having a grid may lie in its ability to coordinate group performance through multiple, "consonant" periodicities (Keller and Burnham 2005).[26] In other words, it may be easier for ensemble members to lock into a common beat that projects itself, in the form of small-integer multiples and divisions, onto other levels of the hierarchy. When onsets avoid the hierarchy, we usually quantize them in order to facilitate coding. But this does not mean that listeners are inherently deprived from experiencing gridless relationships. Katsuyuki Sakai and his colleagues used brain imaging to point to two distinctive neural representations of rhythm: those that employ simple integer ratios and those that do not (Sakai, et al. 1999). Complex ratios elicited neural activation in areas involved in higher order cognitive processes, such as attention and working memory. This was not the case with simple ratios. Since these do not require explicit representation of individual time intervals, they lend themselves to hierarchical organization, thus placing less of a tax on computational resources. Significantly, the two types of rhythmic patterns led to right and left hemispheric dissociation, supporting the view that rhythm processing is more distributed than previously believed. This also suggests that music containing gridless beats engages regions of the brain not normally tapped by the periodic rhythms found in most beat-based music. Perhaps the ideas contained in this article will serve to counterbalance some of the widespread predilections for coordinated periodicity among hierarchical levels.

Notes

1. Joel Lester writes that in Milton Babbitt's music, "without sufficient regularity in any set of impulses, there are too few cues which resonate within a listener to enable him or her to establish a metric grid" (1986, 122). See also London (2004, 24).
2. Interonset intervals faster than 100 ms sound blurred and unmetered. See London (2004, 28–29) for an overview.
3. Stan Getz and Chet Baker, *Stan Meets Chet* (Verve), recorded in 1958.
4. An unfilled point at the end of the graph means that the *y*-value is not known because the note is followed by a rest (as opposed to by another onset, which would allow for the measurement of an IOI).
5. In this and similar graphs, only the beats are sounded by the rhythm section accompaniment. The subdivisions are often implicit.
6. We would still obtain gridlessness if we were to subdivide this excerpt using values reasonably smaller than the sixteenth.
7. The RCA Victor Jazz Workshop, *The Arrangers* (BMG), recorded in 1956.
8. Note that we skipped $s = 4$ because the 3:1 ratio is rarely found in swung melodies. We also skipped $s = 6$ (4+2) because it is reducible to 2+1.
9. Thelonious Monk, *Solo Monk* (Sony), recorded in 1964.
10. Historically, rubato has come in two flavors (Hudson 1994, 1). In the "earlier" type, the tempo remains fixed while the melodic line is rhythmically altered. In the "later" type, the whole musical surface is altered. The present article addresses special cases of the "earlier" type. It goes without saying that the forthcoming discussion can be amended to address ritardandos.
11. John Coltrane, *Blue Train* (Blue Note), recorded in 1957.
12. *The Complete Norman Granz Jam Sessions* (Verve), recorded in 1952.
13. The total degree of gridlessness might be the sum of these individual sums. The size of the individual sums increases towards the

polyrhythm's half-way point and then decreases palindromically; see Folio (1995).

14. Eric Dolphy, *Far Cry* (Prestige), recorded in 1960.

15. I have notated it as three quarter note triplets followed by a quarter note. But in all three instances, the actual timing traces a gradual ritardando from the first quarter note triplet, which is slightly compressed, to the final quarter note.

16. Louis Armstrong and his Hot Five, *Hot Fives & Sevens, Volume 3* (JSP CD 314), recorded in 1928. John Coltrane, *Lush Life* (Prestige), recorded in 1957.

17. Note the embedded accelerations—and decelerations—of the kind discussed earlier.

18. An objective assessment of what constitutes "rhythmic complexity'" or "notational simplicity" is neither necessary for this discussion nor within its scope. I use these terms in their everyday sense. Nauert (1994, 227), who explores some of these same questions, differentiates between performance complexity ("how hard is it?") and listening complexity ("how complex does it sound?").

19. While in principle this technique is polyrhythmic, the effect is perceived more as wavering.

20. Carter's techniques are covered in Bernard (1998) and Schiff (1998); Nancarrow's in Gann (1995). Mead (2007) and Nicolas (1990) explore compositional applications of tempo modulation. I thank Brian Ferneyhough for drawing my attention to the latter article.

21. This table and some of its related ideas are derived from Benadon (2004). These types of equivalences can also be determined with the durational slide ruler cleverly devised by Perkins (1965).

22. Noah Getz, *Crosscurrents* (Albany), recorded in 2007. The saxophone part is notated in E♭.

23. As Mead (2007) points out, electronic music is immune to this caveat because computers face no technical limitations relating to temporal performance. As Babbitt (1984/2003, 386) puts it: "Electronic artifacts have no notational inhibitions or prejudices."

24. Musicians were more sensitive to these "deviant tones" than nonmusicians. See also Patel (2008, Figure 3.3).

25. There is a vast amount of literature on these topics. For excellent reviews and elaborations, see Clarke (1999), Huron (2006, especially 175–202), London (2004), and Patel (2008, especially 99–108).
26. However, there is also evidence to suggest that hierarchical structures do not reduce variability in beat synchronization; see Patel et al. (2005).

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